AB EXAM

Texas A&M High School Math Contest

November 12, 2022

Directions: *All answers should be simplified and if units are involved include them in your answer.*

1. An aquarium has a rectangular base that measures 100 cm by 40 cm and has a height of 50 cm. It is filled with water to a height of 40 cm. A brick with a rectangular base that measures 40 cm by 20 cm and a height of 10 cm is placed in the aquarium. By how many centimeters does the water rise?

2. A train moves at a constant speed. It takes 10 seconds for the entire train to pass a standing observer, and it takes 30 seconds for the entire train to completely cross a bridge 400 meters long. What is the length of the train (in meters)?

3. Eli, Joshua, and Luke are brothers. Eli is 2 years older than Joshua; and Joshua is 1 year less than three times as old as Luke. Together, they are 14 years old. How old is Eli?

4. The numerator and denominator of a fraction are positive integers summing up to 101 and the fraction is not greater then $\frac{1}{3}$. Find the largest possible value of the fraction.

5. Flickering Fred's Christmas lights aren't working. Initially, Fred has 90% of his light bulbs working. After changing some of them in the morning of one day, Fred has 94% of his light bulbs working. He returned to work in the afternoon of the same day and changed 9 less than twice the number of bulbs he changed in the morning. After this Fred has 99% of his light bulbs working. If the only way to make a broken light work is to change the bulb, how many light bulbs does Fred have working when 99% of his lights are working?

6. It is known that $\frac{x+y}{x-y} + \frac{x-y}{x+y} = 3$. Find the value of $\frac{x^2+y^2}{x^2-y^2} + \frac{x^2-y^2}{x^2+y^2}$.

7. Each member of a family drank a full cup of coffee with milk over breakfast (milk is just a usual liquid milk). All cups are identical. It is known that Alice, a member of the family, drank 1/4 of the total amount of milk and 1/6 of the total amount of coffee consumed by the family during this meal. It is assumed that some (nonzero) amount of coffee and some (nonzero) amount of milk was consumed. How many members are in this family?

8. Frolicking Frank decides to take a trip across Texas. Frank starts in El Paso and travels 850 miles to Port Arthur. He starts to travel with certain constant speed. However, making exactly half of the distance Frank realizes that he is running late and decides to increase his speed by 18 miles per hour for the rest of his trip to Port Arthur. It is known that the total time he spent on this travel is the same as he would travel all the way with constant velocity which is 8 miles per hour faster than his initial speed. What was his initial speed?

9. Find the largest x such that
$$\frac{4^{x^2+5}}{8^{4x}} = (0.125)^{3x-5}$$

10. If dividing $x^3 - kx^2 + 5x + 8$ by $x - 1$ yields a remainder of 5, find k.

11. There are two types of trees growing in the park, oaks and maples. At the beginning of this year oaks constituted 60% of all trees. New trees were planted two times during the year, in Spring and in Fall. In Spring only maples were planted and after this procedure the oaks constituted 20% of all trees at the park. In Fall only oaks were planted and after this procedure they again constituted 60% of all trees. Let e be the number of trees in the park at the end of the year and b be the number of trees in the park at the beginning of the year. Find the ratio $\frac{e}{b}$. It is assumed that not a single tree in the park was cut down or fell by itself during the year.

12. The sum of the squares of the roots of the polynomial $2x^2 + x + c$ is equal to 5. Find c.

13. What is the sum of the digits in the number $10^{55} - 55$?

14. The digits of a three-digit number A are reversed and the resulting number is summed with the original number to get 1332. What is the smallest possible value for A?

15. Find a positive four-digit integer which has the decimal representation $(abba)_{10}$ and is a perfect cube.

16. Given p and q are positive integers satisfying the following equality $p! + 12 = q^2$, find $p + q$. (Here $p! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (p-1) \cdot p$, the *factorial of* p.)

17. If $f(1) = 5$ and $f(n+1) = 2f(n) + 1$ for all integers $n \geq 1$, find $f(11)$.

18. Represent the number $(101011011011)_2 + (301021)_4 + (6711)_8$ in the base 8. Here $(x_1x_2\ldots x_k)_b$ denotes the number having digits $x_1, \cdots, x_k$ in base b.

19. 12 numbers are written along the circle. It is known that each number is equal to the absolute value of the differences of two numbers next to it in clockwise direction. It is also known that the sum of these numbers is equal to 1. What is the sum of their cubes?

20. On an exam, Question 4, Question 5, and Question 11 prove to be exceptionally difficult. 38% answered Question 4 correctly and 31% answered Question 5 correctly. 46% of students answered all three questions incorrectly and only 6% of students answered all three questions correctly. There were 18% of students who answered both Question 4 and Question 5 correctly. If 9% of students answered only Question 4 correctly (missing the other two questions) and 13% answered only Question 5 correctly, how many students answered only Question 11 correctly?