TEXAS A&M UNIVERSITY
DEPARTMENT OF MATHEMATICS
MATH FAIR CONTEST
GRADES 8-12

APRIL 22, 2012

Problem 1. Let $f(x) = ax + b$ be a linear function such that $f(4) - f(2) = 5$. Calculate $f(8) - f(2)$.

Problem 2. What accute angle x (measured in degrees) satisfies the equation

$$\sin x = \frac{1}{4\cos x}.$$

Problem 3. What number should come next in the list

$$3,\ 5,\ 9,\ 17,\ 33,\ \ldots$$

Problem 4. I have several stickers and my sister has some of her own. If I gave her one of my stickers she would have the same number of stickers as me. But if she gave me two of hers, I would have three times as many as she has. How many stickers do I have?

Problem 5. Calculate

$$3 - 4 + 5 - 6 + 7 - 8 + 9 - \cdots - 98 + 99.$$

Problem 6. What is the sum of the digits in $2^{2012} \cdot 5^{2016}$?

Problem 7. Let x be a real number such that $x + \frac{1}{x} = 5$. Calculate $x^4 + \frac{1}{x^4}$. (Note that the answer is an integer.)

Problem 8. The school gym needs to have 3 basketballs for every 80 students in the school. If the shool has 560 studens and it already has 10 basketballs, how many basketballs are still needed?

Problem 9. A certain item used to cost 1000 dollars. Its price was then increased by 10 percent, then later reduced by 10 percent and then increased by 10 percent again. What is its current price?

Problem 10. What is the largest possible number of rectangular cards of size 3 cm by 5 cm that can be cut out of a rectangular piece of paper of size 15 cm by 7 cm?

Problem 11. Jane wrote all numbers from 1 to 1000 on the board. How many times did she use the digit 1?

Problem 12. What is the sum

$$\frac{1}{4 \cdot 6} + \frac{1}{6 \cdot 8} + \frac{1}{8 \cdot 10} + \cdots + \frac{1}{2010 \cdot 2012}$$

Write the answer as a single fraction in reduced form.

Problem 13. A 4-digit number is given. If we move the last digit, which is equal to 4, to the first position we obtain a new number that is smaller than the given number by 1107. What is the given number?

Problem 14. There are 5 blue, 4 green and 3 red balls in a jar. Susan randomly takes three balls, one by one, out of the jar (she does not return the balls once she takes them out). What is the chance (probability) that the first two balls are blue, but the third one is not? (You can leave the answer as a product of fractions).

Problem 15. In a certain month, there were three Sundays on even dates (the day of the month was even). What day (Monday, Tuesday, ...) was the 14th day of that month?

Problem 16. One day John said "two days ago I was 8 years old, and next year I will be 11". When is John's birthday (what date)?

Problem 17. Bob wanted to buy three video games A, B and C. However, he realized that he only had enough money for two. The total price without Game A was 84 dollars, without Game B was 74 dollars and withouth Game C was 70 dollars. What is the cost of Game B?

Problem 18. A regular 6-sided die is rolled and the product P of the 5 visible numbers (all but the one on which the die rests) is calculated. What is the largest natural number that is certain to divide P in every case?

Problem 19. A certain function f satisfies the equality $f(4-x) = f(4+x)$, for every real number x. Moreover, the equation $f(x) = 0$ has 6 distinct solutions. What is the sum of these solutions?

Problem 20. In the triangle $\triangle ABC$, the lengths of the three sides are $AB = 5$, $BC = 7$ and $AC = 9$. Let D be the point on AC, different from A, for which $BD = 5$. What is the distance from A to D?