Texas A&M University - Department of Mathematics Math Fair Contest - April 20, 2013 Grades 7 - 9

Problem 1. Calculate (b-2a)c if a=-3, b=5, c=-2.

Problem 2. Calculate $\sqrt{5} \cdot \left(\sqrt{5}^{\sqrt{5}}\right)^{\sqrt{5}}$.

(In order to count, the answer needs to be simplified as much as possible.)

Problem 3. Which number is one-ninth of 27⁹? (Write the answer as 3 to some power.)

Problem 4. What is $\sqrt{222}$ rounded to the nearest integer?

Problem 5. A wooden cube is painted in maroon on the outside and then cut into 64 equal cubes. The sides of the 64 cubes that are not painted in maroon are then painted in white. What is the ratio of the area painted in maroon and the area painted in white?

Problem 6. A bag contains 6 white and 4 maroon balls. Jessica draws balls at random from the bag (she does not return the balls once she draws them) and she stops as soon as she draws a maroon ball. What is the probability that she stops immediately after the fourth draw? (Provide the answer as a fraction in reduced form.)

Problem 7. There are only two prime numbers between 1 and 50 that have sum of digits equal to 10. What is their difference?

Problem 8. What is the area of the triangle with vertices A with coordinates (1,2), B with coordinates (4,3) and C with coordinates (4,6)?

Problem 9. What is the length of the longest side of the triangle in the previous problem?

Problem 10. The sum of the interior angles of some polygon is 1260 degrees. How many sides does the polygon have?

Problem 11. An isosceles triangle has base length 10 and area 60. How large are its legs?

Problem 12. Jane wrote all numbers from 10 to 200 on the board, then she deleted all numbers that did not have the digit 2 as the second to last digit. What is the average of the numbers that were left on the board?

Problem 13. Let A be the number obtained by multiplying all numbers from 1 to 100. The number A ends in several zero digits. How many?

Problem 14. In a certain middle school, seventh graders that are in the orchestra practice their instruments an average of 30 minutes a day. If there are 40 seventh graders and 60 eight graders in the orchestra, and the average practice time per day for all students is 42 minutes, how many minutes per day do the eight graders in the orchestra practice on average?

Problem 15. I have several stickers and my sister has some of her own. Our friend has twice as many stickers as the two of us have together. If I got one more sticker our friend would have three times as many stickers as I would. If my sister got another sticker our friend would have four times as many stickers as she would. How many stickers does our friend have?

Problem 16. The electricity went out and it is pitch-dark in your room (you cannot see anything). You are packing for a trip and you know that you have 6 white, 8 black, and 12 maroon socks in your drawer. What is the smallest number of socks you need to pack in your bag to make sure that you have packed at least one matching pair of socks?

Problem 17. At some moment between 4 and 5 o'clock John looked at the clock and noticed that the large hand is exactly two minutes ahead of the small hand on the dial. What was the time at that moment?

Problem 18. Starting with the number 15, suppose you perform the following four operations: add 3, subtract 3, multiply by 3, and divide by 3, one after another, each exactly once, but not necessarily in that order. What is the largest number you can end up with?

Problem 19. A square of size 1×1 is cut out of the corner of a cardboard square of size 8×8 . The part that remained after the small square was thrown was then cut into some number of triangles all of which have the exact same shape and size. What is the smallest possible number of such triangles?

Problem 20. For what value of a does the equation

$$|||x| - 5| - 3| = a$$

have odd number of solutions?