Texas A&M Mathematics Department Mini Fair Math Contest

Grades 8-12

April 16, 2011

Please write your answers on the provided solutions page.

1. What is the sum of the first 100 even numbers.

2. Which statement is true about $\left(\frac{10}{11}\right)^{111} \cdot \left(\frac{11}{10}\right)^{211}$?

   a. The product is greater than 1,000.
   b. The product is greater than 700 but less than 1000.
   c. The product is greater than 3 but less than 700.
   d. The product is greater than 1 but less than 3.
   e. The product is less than 1 but greater than 0.

3. Solve the equation $|x-3| + |x-6| + |x-9| = 12$.

4. Evaluate the product $(\log_2 3)(\log_3 4)(\log_4 5)(\log_5 6)(\log_6 7)(\log_7 8)$.

5. Evaluate $\sec(\frac{5\pi}{12})$.

6. In a triangle the side lengths are 12, 16, and 18. Is the triangle acute, right, or obtuse?

7. Find $\tan C$, where C is the angle opposite to side c of a triangle whose side lengths a, b, and c satisfy

$$\frac{a^3 + b^3 + c^3}{a + b + c} = c^2.$$

8. Given the points $A = (1, 1.5)$ and $B = (4, 5)$ find the minimum distance $|AC| + |CB|$ where C is a point on the line $y = 1 - 2x$.

9. Find the number of zeros at the end of 2011!

10. Simplify the expression

$$\sqrt{6 + \sqrt{11}} - \sqrt{6 - \sqrt{11}}.$$

11. Find $\max(\min(\{x, y\}))$ if the points (x, y) have nonnegative coordinates and are on the line $x + y = 16$.

12. An integer is chosen at random from the set $\{x \,|\, 1 < x \leq 501\}$. Find the probability that this integer is divisible by 7 or 11. Your answer should be a single

fraction in reduced form.

13. If $\log_y x + \log_x y = 8$, then find the value of $(\log_y x)^2 + (\log_x y)^2$.

14. Let $f(x)$ be a function such that $f(x) + 2f(-x) = \cos x$ for every real number x. What is the value of $f(\pi)$? Your answer should be a single fraction in reduced form.

15. Solve the equation $2^x - 2^{-x} = \sqrt{5}$.

16. There are ten different science books on the same shelf. Four of them are about mathematics and the rest are physics books. Find the probability of the event that all mathematical books are next to each other and all physics books are next to each other on the shelf. Your answer should be a single fraction in reduced form.

17. Determine the number of solutions of the equation $\cos(x) = \dfrac{x}{2011}$ on the interval $[\pi, 16\pi]$.

18. Solve the system of equations
$xy = 12\sqrt{6}$
$yz = 54\sqrt{2}$
$zx = 48\sqrt{3}$.

19. Find the limit of the sequence $\sqrt{2}, \sqrt{1+\sqrt{2}}, \sqrt{1+\sqrt{1+\sqrt{2}}}, \ldots$.

20. Solve the equation

$$(x+1)(x+2)(x+3)(x+4) + 1 = 0.$$