PROPOSED CONTENT OF THE QUALIFYING EXAM ON NUMERICAL ANALYSIS

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1. Numerical linear algebra

(Stoer & Bulirsch, Cheney & Kincaid, Golub & Van Loan)

1. Gauss and Jordan eliminations, matrix inversion, pivoting strategy, LU and Cholesky decompositions.
2. Vector and matrix norms. Condition number and its connection to the stability of the solution of algebraic systems.
3. Eigenvalues and eigenvectors of matrices (minimax methods for symmetric matrices, power method, QR method). Singular value decomposition and its basic properties.
4. Symmetric and positive definite matrices and their properties.
5. Iterative methods for linear systems (Jacobi, Gauss-Seidel, SOR and the idea of preconditioning).

2. Fundamentals of numerical analysis

(Stoer & Bulirsch, Cheney & Kincaid)

1. Polynomial and spline interpolation and least-squares approximation. (Lagrange and Hermite interpolation formulas and their errors, Neville's algorithm, Newton's divided difference formula, quadratic and cubic splines, least-squares).
2. Numerical differentiation and integration - interpolatory and Gauss quadratures, extrapolation and adaptive integration. (trapezoidal rule, Simpson's rule, closed and open Newton-Cotes formulas, Peano kernel theorem, orthogonal polynomials, Richardson extrapolation, error estimates).
3. Iteration methods for nonlinear equations. (Bisection algorithm, fixed point iteration, Newton method, secant method, order of convergence).

3. Initial value problems for ordinary differential equations

(Stoer & Bulirsch, Cheney & Kincaid)

1. Methods for initial value problems for ordinary differential equations: Runge-Kutta and Adams methods.
2. Methods with automatic step-size control for Runge-Kutta and Adams methods.
3. Basic concepts of stability of the multistep methods for ODE's and systems.

4. Boundary value problems for ordinary differential equations

(Stoer & Bulirsch, Cheney & Kincaid, Ames, Johnson)

1. Finite difference and finite volume approximations.
2. Weak formulations and finite element Gelerkin method.
3. Stability and error estimates: maximum principle, energy type estimates and matrix stability.
4. Approximation, stability and convergence.

5. Numerical methods for parabolic problems

(Ames, Striktwerda, Johnson)

1. Finite difference approximations: explicit, implicit and Crank-Nicolson schemes.
2. Stability: maximum principle, Fourier mode analysis, matrix stability and energy type estimates (Courant condition).
3. Error estimates

6. Numerical methods for elliptic problems

(Ames, Striktwerda)

1. Finite differences and finite volumes: approximation of the equation and the boundary conditions, higher order schemes.
2. Stability and error analysis: maximum principle, Fourier analysis, energy type estimates.
3. Iterative methods for approximations of elliptic problems: Jacobi, Gauss-Seidel and SOR and their convergence rates.

7. Finite element method

(Johnson, Ciarlet, Strang & Fix)

1. Weak (variational) formulation and characterization of the energy space: essential and natural boundary condition.

2. Ritz-Galerkin method.

3. Finite element method (linear and quadratic triangles and bilinear and biquadratic rectangles).

4. Error estimates, Bramble-Hilbert lemma, Nitsche trick.

5. Galerkin finite element method for transient problems.

8. References

1. J. Stoer and R. Bulirsch, Introduction to Numerical Analysis, Second Edition, Springer-Verlag, 1993.

2. W. Cheney and D. Kincaid, Numerical Analysis,

3. W. Ames, Numerical Methods for PDE's, Third edition, Academic Press, 1992.

4. C. Johnson, Numerical Solutions of PDE's by the Finite Element Method, Cambridge University Press, 1987.

5. J. C. Striktwerda, Finite Difference Schemes and PDE's, Wadsworth & Brooks, 1989.

6. Ph. Ciarlet, The Finite Element Method for Elliptic Problems, North-Holland, 1978 (paperback, 1980).

7. G. Strang and G. Fix, An Analysis of the Finite Element Method, Prentce Hall, Englewood Cliffs, N.J., 1973.

8. G. H. Golub and C. van Loan, Matrix Computations, John Hopkins University Press, Baltimore and London, Second Edition, 1989.