2018 Texas A&M REU Miniconference

July 16, Blocker Building, Room 220

SCHEDULE

MONDAY, July 16

08:00–08:25	Breakfast snacks in Blocker 220/246	Bluebaker
08:30–08:50	Geometry of Real Roots, with an Eye Toward Chemical Reaction Networks I	Lacey Eagan
08:55–09:15	Geometry of Real Roots, with an Eye Toward Chemical Reaction Networks II	Luis Feliciano
09:20–09:40	A Faster Randomized Algorithm for Counting Roots in $\mathbb{Z}/(p^k)$ I	Natalie Randall
09:45–10:05	A Faster Randomized Algorithm for Counting Roots in $\mathbb{Z}/(p^k)$ II	Lean Kopp
10:20–10:40	Integral Metapletic Modular Categories I	Leslie Mavrakis
10:40–10:50 **11:00–11:10**	Integral Metapletic Modular Categories II	Sydney Timmerman
11:10–11:30	Integral Metapletic Modular Categories III	Benjamin Warren
11:35–11:55	On classification of modular tensor categories	David Green
12:00–12:55	Lunch in Blocker 246	Taz
12:55–13:15	Effective bounds for traces of singular moduli I	Meagan Kenney
13:15–13:35	Effective bounds for traces of singular moduli II	Havi Ellers
13:40–14:00	Dedekind Sums Arising from Generalized Eisenstein Series I	Tristie Stucker
14:05–14:25	Dedekind Sums Arising from Generalized Eisenstein Series I	Amy Vennos

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ABSTRACTS

(In order of appearance)

Geometry of Real Roots, with an Eye Toward Chemical Reaction Networks I,
by Lacy Eagan (Howard Payne University, Brownwood, Texas)

Chemical reactions are constantly taking place within every single animal and plant cell, and these reactions can be modeled by Chemical Reaction Networks. The particular chemical reaction that we chose to study through Chemical Reaction Networks is Phosphorylation. After modeling Phosphorylation, we use Mass Action Kinetics to derive nine reaction rate equations with nine unknowns which we can then solve to find the equilibria of our Chemical Reaction Network. We are interested in the equilibria because we would like to see when they take place and under what conditions. After reducing our 9×9 system to a much simpler 2×2 system, we apply $\mathcal{A}$-Discriminants to obtain a better idea of what our roots will look like. Discriminant Varieties divide the coefficient space into regions where the underlying number of real roots is constant, and we use Linear Programming to see which coefficient sign vectors yield simpler root counting.

Geometry of Real Roots, with an Eye Toward Chemical Reaction Networks II,
by Luis Feliciano (New York, New York)

Solving for the real roots of polynomial systems becomes more challenging as we add more terms and variables. However, we can use convex geometry to find metric estimates of real roots in a fraction of the time! Here we will develop some of the tools necessary to build up to our main tool — the Archimedean Tropical Variety, a piece-wise linear construction guaranteed to be close to our complex zero set, and a signed variant that has the same isotopy type as the positive zero set with high probability. We will show the results of applying these constructions to a family of 9×9 polynomial systems arising from chemical reaction networks.

A Faster Randomized Algorithm for Root Counting in $\mathbb{Z}/(p^k)$ I,
by Natalie Randall (Austin College, Sherman, Texas)

Given a univariate polynomial $f \in \mathbb{Z}/(p^k)$, we can write f as a Taylor expansion using the following perturbation: Let ζ be some root of f in $\mathbb{Z}/(p)$ and let ε be some value in $Z/(p^{k1})$; then we can consider

$$f(\zeta + p\varepsilon) = f(\zeta) + f'(\zeta)p\varepsilon + \frac{1}{2!}f''(\zeta)p^2\varepsilon^2 + \cdots + \frac{1}{(k-1)!}f^{(k-1)}(\zeta)p^{k-1}\varepsilon^{k-1} \bmod p^k.$$

By dividing by suitable powers of p and reducing this expansion modulo lower powers of p, we can recursively isolate the roots of f in the ring $\mathbb{Z}/(p^k)$. We thus attain a Las Vegas randomized complexity bound of $d^{1.5+o(1)}(\log p)^{2+o(1)}1.12^k$ (apparently the fastest to date) for counting the roots of f in $\mathbb{Z}/(p^k)$.

A Faster Randomized Algorithm for Root Counting in $\mathbb{Z}/(p^k)$ II,
by Leann Kopp (Auburn University, Auburn, Alabama)

Recent work by Cheng, Gao, Rojas, and Wan introduced a deterministic algorithm for counting the number of roots of a univariate polynomial f in $\mathbb{Z}/(p^k)$, where p is a prime, k a positive integer, and f is not identically 0 mod p. Here we discuss a new randomized algorithm for this problem, and compare our new algorithm to simple brute-force to see when we have practical time gains. In addition, we present an upper bound on the number of roots of f (as a function of p, k, and the degree of f) that is optimal for $k = 2$.

Integral Metapletic Modular Categories I, II, and III,
by Leslie Mavrakis (Seattle Pacific University, Seattle, Washington), Sydney Timmerman (Johns Hopkins University, Baltimore, Maryland), and Benjamin Warren (Swarthmore College, Swarthmore, Pennsylvania)

At the intersection of mathematics, physics, and computer science, topological quantum computation utilizes the swapping of non-Abelian anyons to perform computation that is topologically protected against decoherence. This swapping gives a representation of the braid group, where the strands are the anyon world lines. If the image of the braid group representation is dense, quantum gates can usually be approximated to arbitrary accuracy by braiding anyons. Modular categories provide one model for anyonic systems; in this model, each simple object corresponds to one anyon type, and the fusion rules defined on each pair of simple objects allow the fusion of two anyons into a direct sum (superposition) of other anyons. From each modular category a braid group representation can be constructed. We specifically study integral metapletic modular categoriesmodular categories with integer dimension that have the same fusion rules as the quantum group category $\mathrm{SO}(N)_2$. We prove integral metaplectic modular categories are group theoretical, which implies these categories have Property F, i.e. the associated braid group representations have finite image and are necessarily non-universal. This means that anyonic systems with these fusion rules cannot be used to create a universal topological quantum computer using only braiding. The extended Property F conjecture also suggests that there is a classical link invariant associated with each of these categories. Beginning with a special case, we attempt to determine this link invariant.

On classification of modular tensor categories,
by David L. Green (U Texas, Austin, Texas)

Modular Tensor Categories (MTCs) arise in the study of certain condensed matter systems. There is an ongoing program to classify MTCs of low rank, up to modular data. We present an overview of the methods to classify modular tensor categories of low rank, applied to the specific case of a rank 6 category with Galois group $\langle(012)(345)\rangle$, and give evidence that certain symmetries in this case imply nonunitarizable (hence, nonphysical) MTCs.

Effective bounds for traces of singular moduli,
by Havi Ellers (Harvey Mudd College, Claremont, California) and Meagan Kenney (Bard College, Annandale-on-Hudson, New York)

Values of the classical modular j-function at CM points are algebraic integers called singular moduli. They play an important role in number theory. In this talk, we will explain how to give effective upper bounds for traces of singular moduli using Poincare series and harmonic analysis on the complex upper half-plane.

Dedekind Sums Arising from Generalized Eisenstein Series,
by Tristie Stucker (University of Idaho, Moscow, Idaho) and Amy Vennos (Salisbury University, Salisbury, Maryland)

Given primitive Dirichlet characters χ_1 and χ_2, we study the weight zero Eisenstein series $E_{\chi_1,\chi_2}(z,s)$ at $s=1$. We examine transformation properties of terms arising from the Fourier expansion of the Eisenstein series, and we express these properties with a generalized Dedekind sum formula in terms of Bernoulli functions.