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“On Connectedness and Perturbations of the Constant-Diagonal Idempotents”

An $n \times n$ matrix A is said to be idempotent if $A^2 = A$. We consider the set of $2n \times 2n$ idempotents over \mathbb{C} with $\text{rank}(A) = n$ and all diagonal entries equal to $\frac{1}{2}$. We characterize the possible ranges of idempotents with diagonal $\frac{1}{2}$, which turns out to be an open, dense, connected subset of the set of all n -dimensional subspaces of \mathbb{C}^{2n} . Moreover, for any idempotent with diagonal entries sufficiently close to $\frac{1}{2}$, there exists an idempotent with constant diagonal and the same range. These results are motivated by a problem concerning the connectedness of such idempotents.