Title: MATH 666 Positive Scalar Curvature

Course Description: This course explores the rich interplay between positive scalar curvature (PSC) metrics and the geometry and topology of the underlying manifolds through three interconnected parts:

1. Positive Scalar Curvature and Bordism:
   - We begin with the classical surgery theorem of Gromov-Lawson and Schoen-Yau, which establishes that PSC metrics are preserved under surgeries of codimension at least three. Building on this foundation, we examine Stolz's theorem, which provides a complete classification of PSC metrics on simply-connected manifolds of dimension at least five.

2. Minimal Hypersurface Method:
   - The second part introduces the Schoen-Yau minimal hypersurface method and its profound implications. Using this technique, we will revisit the resolution of the Geroch conjecture, highlighting its significance in scalar curvature geometry.

3. Potentials and μ-bubbles:
   - The final part focuses on the role of potentials and the concept of μ-bubbles—hypersurfaces with prescribed mean curvature, derived by minimizing the volume-functional perturbed by a potential function. Applications include recent results such as:
      a. the resolution of the Gromov-Lawson conjecture in dimensions 4 and 5, asserting that closed aspherical manifolds admit no metrics of PSC.
      b. The solution to Rosenberg's S^1-stability conjecture up to dimension 7, which states that a closed n-dimensional manifold M (for n≠4) admits PSC if and only if M×S^1 does.

This course combines classical results with recent advancements, equipping participants with the tools and insights to understand current research questions in scalar curvature geometry.