Title: MATH 664 Multi-scale Analysis in Fluid Dynamics Equations

Course Description: This is an introductory course to the rigorous analysis of multi-scale features in fluid dynamics equations, with the emphasis on flows in geophysics fluid dynamics.
The goal is to introduce the principle of rigorous multi-scale analysis through some classical examples, including the low Mach number limit (incompressible limit) of compressible fluid,
fast rotating limit of incompressible fluid, and quasi-geostrophic limit of the Boussinesq equations, etc.. Students are expected to be able to establish the formal asymptotic expansion,
as well as rigorous justification, of these classical problems, and will be able to pinpoint the key steps in the multi-scale analysis.

Structure of the course:

- Singular limit in systems of ordinary differential equations;
- Well-posedness theory of short-time regular solutions to hyperbolic systems;
- Brief introduction to the fluid dynamics equations;
- Steve Schochet’s fast oscillation theory, singular limit in periodic domains;
- Strichartz estimates for wave equations, singular limit in R^n;
- Zero Mach number limit for isentropic compressible flows in periodic domains and R^n;
- Zero Rossby number limit for rotating flows in periodic domains;
- Quasi-geostrophic limit for the Boussinesq equations in a bounded channel.