Course Information

Course Number:	MATH 689
Course Title:	Elliptic Curves
Section:	6xx
Time:	TBD
Location:	TBD
Credit Hours:	3

Instructor Details

Instructor:	Matthew Papanikolas
Office:	Blocker 641E
Email:	papanikolas@tamu.edu
Office Hours:	TBD

Course Description

Elliptic curves make up an important class of geometric objects, which have connections in many areas of mathematics, especially number theory and algebraic geometry, as well as several applications, including to cryptography and integer factorization. At the outset elliptic curves are merely solutions of certain cubic polynomial equations, but the points on these curves possess a natural abelian group structure, which leads to many mesmerizing problems.

This course will focus on the study of elliptic curves over various fields, including the rational numbers, number fields, finite fields, the p-adic numbers, and the complex numbers. The two main goals of the course will be to prove the Mordell-Weil theorem, which states that the group of points over a number field (including the rational numbers) is finitely generated, and to discuss the Birch and Swinnerton-Dyer conjecture, which relates the rank of the Mordell-Weil group to the vanishing of the L-function of the elliptic curve.

The course will cover the following topics as time permits:

- Algebraic curves and the Riemann-Roch theorem
- Elliptic curves as plane curves, Weierstrass models
- The group law on an elliptic curve
- Torsion points
- Isogenies and the Tate module
- Formal groups, Hensel's lemma
- Elliptic curves over finite fields
- Hasse's bound on points over finite fields
- Congruence zeta functions and the Riemann hypothesis over finite fields
- Elliptic functions and elliptic curves over the complex numbers
- Elliptic curve cryptography

- Reduction modulo primes (good, bad, and not so bad)
- Height functions
- The Mordell-Weil theorem
- The Hasse-Weil L-function and the Birch and Swinnerton-Dyer conjecture

Course Prerequisites

The course prerequisites are Math 653/654 (Graduate Algebra I & II), or consent of the instructor. Otherwise, the course will be fairly self-contained, and necessary elements of algebra, number theory, and algebraic geometry will be covered during the semester.

Students interested in doing research in Number Theory are especially encouraged to attend this course.

Special Course Designation

N/A

Course Learning Outcomes

Upon successful completion of this course, students will be able to

- Define elliptic curves as algebraic curves of genus one with a fixed based point
- Apply the Riemann-Roch Theorem to a wide range of problems on algebraic curves
- Transform elliptic curves into standard Weierstrass form
- Develop the group law on sets of rational points on an elliptic curve
- Determine the structure of the torsion points on an elliptic curve
- Identify isogenies and their duals between curves
- Define the Tate module and its associated Galois representations
- Prove the Weil Conjectures/Hasse's Theorem for elliptic curves over finite fields
- Determine primes of bad reduction and their reduction types for elliptic curves over p-adic fields
- Develop the theory height functions on number fields
- Prove the Mordell-Weil Theorem for elliptic curves over the rational numbers and more generally over number fields

Textbook

Required Textbooks:

The Arithmetic of Elliptic Curves, 2nd Ed., J. H. Silverman, Springer, Dordrecht, 2009, 9780387094939.

Homework

Homework assignments will be announced in class or posted in Canvas periodically. Turn in in class or by email.

Presentations

There will be an opportunity for students to present material to the class at the end of the semester on a topic of their choice. These presentations will be approximately 15 minutes. **Giving a presentation is optional,** and if you would like more information about what it might entail, let me know.

Grading Policy

Student grades will be determined by performance on **homework assignments** and optional **presentation**. The following grade distribution will be used in determining final course grades:

Range	Grade
90.0%-100.0%	A
80.0%-89.9%	B
70.0%-79.9%	C
60.0%-69.9%	D
0.0%-59.9%	F

Late Work Policy

Excused absences: For excused absences we refer students to Student Rule 7 at https://student-rules.tamu.edu/rule07/. Excuses for absences during an exam must be substantiated by appropriate documentation.

Make-up homework and exams will only be allowed due to excused absences and the timeline must be discussed with the instructor, following Student Rules. If you foresee the need to be absent during an exam, you must notify the instructor in advance.

University Policies

Attendance Policy

The university views class attendance and participation as an individual student responsibility. Students are expected to attend class and to complete all assignments.

Please refer to Student Rule 7 in its entirety for information about excused absences, including definitions, and related documentation and timelines.

Makeup Work Policy

Students will be excused from attending class on the day of a graded activity or when attendance contributes to a student's grade, for the reasons stated in Student Rule 7, or other reason deemed appropriate by the instructor.

Please refer to Student Rule 7 in its entirety for information about makeup work, including definitions, and related documentation and timelines.

Absences related to Title IX of the Education Amendments of 1972 may necessitate a period of more than 30 days for make-up work, and the timeframe for make-up work should be agreed upon by the student and instructor" (Student Rule 7, Section 7.4.1).

"The instructor is under no obligation to provide an opportunity for the student to make up work missed because of an unexcused absence" (Student Rule 7, Section 7.4.2).

Students who request an excused absence are expected to uphold the Aggie Honor Code and Student Conduct Code. (See Student Rule 24.)

Academic Integrity Statement and Policy

"An Aggie does not lie, cheat or steal, or tolerate those who do."

"Texas A&M University students are responsible for authenticating all work submitted to an instructor. If asked, students must be able to produce proof that the item submitted is indeed the work of that student. Students must keep appropriate records at all times. The inability to authenticate one's work, should the instructor request it, may be sufficient grounds to initiate an academic misconduct case" (Section 20.1.2.3, Student Rule 20).

You can learn more about the Aggie Honor System Office Rules and Procedures, academic integrity, and your rights and responsibilities at aggiehonor.tamu.edu.

Americans with Disabilities Act (ADA) Policy

Texas A&M University is committed to providing equitable access to learning opportunities for all students. If you experience barriers to your education due to a disability or think you may have a disability, please contact the Disability Resources office on your campus (resources listed below) Disabilities may include, but are not limited to attentional, learning, mental health, sensory, physical, or chronic health conditions. All students are encouraged to discuss their disability related needs with Disability Resources and their instructors as soon as possible.

Disability Resources is located in the Student Services Building or at (979) 845-1637 or visit disability.tamu.edu.

Title IX and Statement on Limits to Confidentiality

Texas A&M University is committed to fostering a learning environment that is safe and productive for all. University policies and federal and state laws prohibit gender-based discrimination and sexual harassment, including sexual assault, sexual exploitation, domestic violence, dating violence, and stalking.

With the exception of some medical and mental health providers, all university employees (including full and part-time faculty, staff, paid graduate assistants, student workers, etc.) are Mandatory Reporters and must report to the Title IX Office if the employee experiences, observes, or becomes aware of an incident that meets the following conditions (see University Rule 08.01.01.M1):

- The incident is reasonably believed to be discrimination or harassment.
- The incident is alleged to have been committed by or against a person who, at the time of the incident, was (1) a student enrolled at the University or (2) an employee of the University.

Mandatory Reporters must file a report regardless of how the information comes to their attention – including but not limited to face-to-face conversations, a written class assignment or paper, class discussion, email, text, or social media post. Although Mandatory Reporters must file a report, in most instances, a person who is subjected to the alleged conduct will be able to control how the report is handled, including whether or not to pursue a formal investigation. The University's goal is to make sure you are aware of the range of options available to you and to ensure access to the resources you need.

Students wishing to discuss concerns in a confidential setting are encouraged to make an appointment with Counseling and Psychological Services (CAPS).

Students can learn more about filing a report, accessing supportive resources, and navigating the Title IX investigation and resolution process on the University's Title IX webpage.

Statement on Mental Health and Wellness

Texas A&M University recognizes that mental health and wellness are critical factors that influence a student's academic success and overall wellbeing. Students are encouraged to engage in healthy self-care by utilizing available resources and services on your campus.

Students who need someone to talk to can contact Counseling & Psychological Services (CAPS) or call the TAMU Helpline (979-845-2700) from 4:00 p.m. to 8:00 a.m. weekdays and 24 hours on weekends. 24-hour emergency help is also available through the 988 Suicide & Crisis Lifeline (988) or at 988lifeline.org Links to an external site.